# MixCoin: Anonymity for BitCoin with Accountable Mixes 

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## BitCoin

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- Users have pseudononymous addresses
- Transactions can often be easily linked
- If one transaction is linked to user, then all of their addresses may be exposed


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- Blockchain is a decentralized, publicly verifiable ledger
- Records all past messages exchanged between users on that Blockchain
- No transactions are truly anonymous, because they are always publicly visible on the blockchain


## Background: BitCoin Mixes

Used to preserve privacy for some BitCoin users. Multiple clients send coins to a mixing address, which forwards them randomly to a fresh address for each client.

## Mixing Services

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- Take a user's coins and randomly exchange them for other user's coins
- Obfuscates ownership

Alice owns $N$ bitcoins at address $\kappa_{\text {in }}$, which is linkable to her real-world identity. She wishes to tranfer her funds to address $\kappa_{\text {out }}$ in a way which is difficult to link to $\kappa_{\text {in }}$ for a fee. She sends her funds to a mix $M$ which holds them for an agreed time period before sending an equal value to $\kappa_{\text {out }}$.


## The Downside of Mixes

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- Open question of linking attacks between inputs and outputs


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No protection from theft!

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- Alice could falsely accuse the mix of theft to undermine its reputation
- Accusations of theft cannot be proven, hence it is difficult to determine which mixes are honest
- A malicious mix is able to link the in and out addresses, potentially undermining Alice's anonymity


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- Substantial modifications to BitCoin
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- Entirely New Currency
- CoinJoin, CoinSwap
- Backwards-compatible with BitCoin
- Practical complications, smaller anonymity sets


## Solution: MixCoin!

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- Add cryptographic accountability layer


## MixCoin: Accountability

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- Alice can publish this warranty if the mix fails to deliver her coin


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- MixCoin uses randomized, all-or-nothing fees


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- Single-use mix addresses
- Passive adversaries can't determine which mix a user is interacting with
- Anonymity set: All users who are interacting with any mix at the same time


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- Chains multiple mixes together
- Provides strong anonymity against an active attacker who can break mix indistinguishability


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- Can be deployed immediately on top of BitCoin


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## MixCoin: The Idea

- MixCoin is a protocol for mixing with accountability
- The mix gives Alice a signed warranty which she can use to unabiguously prove that the mix has misbehaved
- There is no way to prove that a mix is not storing records which could deanonymize its clients
- Alice can send her coins through a series of mixes which all must collude to deanonymize her final address


## Assumptions

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- Mix $M_{i}$ represented by warranty-signing key $K_{M_{i}}$
- Each mix's warranty-signing key is used consistently
- Alice able to negotiate with mix over an anonymous, confidential channel (Tor hidden service)


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- $n$, a nonce used to determine payment of randomized mixing fees


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- $w$, the number of blocks the Mix requires to confirm Alice's payment


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- $w=6$ is a common standard


## Step 1

Alice contacts Mix over a secure channel and proposes the mixing parameters

## Step 2

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1. The mix accepts these terms, generates a fresh escrow address $\kappa_{e s c}$, and sends back a warranty containing all of Alice's parameters plus $\kappa_{\text {esc }}$
2. The mix rejects Alice's request

Note that $\kappa_{\text {out }}$ and $\kappa_{\text {esc }}$ should be fresh addresses created specifically for mixing

## Step 3

Alice transfers the value $v$ to $\kappa_{\text {esc }}$ by time deadline $t_{1}$

## Step 4

1. The mix transfers an equal value to $\kappa_{\text {out }}$ by time $t_{2}$

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2. The mix fails to transfer $v$ to $\kappa_{\text {out }}$ by time $t_{2}$

## Step 5

1. If the protocol is successful, $A$ and $M$ destroy their records

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2. If Alice detects theft, she publicizes

$$
\left\{v, t_{1}, t_{2}, w, \kappa_{\text {esc }}, \kappa_{\text {out }}, \rho, n\right\}_{K_{M}}
$$

## Mixing Fees

Fixed mixing fees undermine the goal of indistinguishable transfers and limit the anonymity set

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- Expected mixing rate is $\rho$


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- This computation can be performed by anybody if Alice's warranty is published (hence cheating is detectable!)


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- Also utilizes the nonce $n$ specified by Alice, used to solove the proof-of-work puzzle


## Beacon from BitCoin blocks

The mix computes

$$
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- PRNG is a cryptographic pseudo random function which outputs a value uniformly drawn from the range $(0,1)$
- $B_{i}$ is the Merkle root of block $i$ in the BlockChain


## Merkle Trees

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Figure: From Wikipedia

## The MixCoin Protocol



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$\kappa_{\text {esc }}^{\prime} \quad \kappa_{\text {esc }}^{*}$
$\kappa_{\text {out }}$
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- Alice obtains $N$ signed warranties, transfers funds to $\kappa_{e s c_{1}}$


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- Each chunk should travel through an independently-chosen random mix of sequences.


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- Expected value of absconding is $\mathrm{E}[$ abscond $]=Q \bar{t}$


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- Expected payoff from choosing to continue defined recursively
- Under steady state conditions, optimal decision the same in every round
- If mix chooses to continue, it will do so indefinitely
- Mix is discounting future earnings at a rate of $r$ per block
- Net value of indefinite honest behavior:

$$
\frac{\rho Q}{r}
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- Let $r$ be equivalent to the highest available risk-free rate of return available
- Then, all this says is that the expected value of fees collected by a mix during the time it holds funds is greater than the amount those funds would yield during the same time period if invested


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- If $r \approx 20 \%$ available to mix , then a mix time $\bar{t} \approx 1$ hour yields lower bound $\rho_{\text {min }} \approx 2^{-15}$
- A chunk taking a path through 10 consecutive mixes leaves a fee rate of $\approx 2^{-12}$


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- Best case scenario: Passive Adversary
- Say adversary can determine with high probability which Bitcoin transactions are mix traffic
- Adversary still may not be able to link escrow addresses to specific mixes due to their one-time nature
- This property is called mix indistinguishability


## Anonymity Properties: Active Adversary

First attack:

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- The client which sends the chunk to $\kappa_{\text {esc }}^{\prime}$, eventually sent to $\kappa_{\text {out }}$, learns that Alice interacted with $M$
- Active adversary can exploit this in a flooding attack


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- If mixes pay transaction fees, then $M$ may use a fee retained from a user to pay a the transaction fees
- All of these transaction fees can then be linked to $M$


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- If she mixed those chunks sufficiently, they will have the same anonymity set
- If even one chunk travels through a path consisting entirely of comprimised mixes, her entire payment loses anonymity
- If $25 \%$ of mixes are comprimised, there is a $2^{-20}$ chance of routing a chunk through a chain of ten comprimised mixes


## Acknowledgements

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## Bibliography

Mixcoin: Anonymity for BitCoin with Accountable Mixes, Joseph Bonneau, Arvind Narayanan, Andrew Miller, Jeremy Clark, Joshua A. Kroll, Edward W. Felten. International Financial Cryptography Association 2014, LCNS 8437, pp.486-504.

